

NAG C Library Function Document

nag_zpocon (f07fuc)

1 Purpose

nag_zpocon (f07fuc) estimates the condition number of a complex Hermitian positive-definite matrix A , where A has been factorized by nag_zpotrf (f07frc).

2 Specification

```
void nag_zpocon (Nag_OrderType order, Nag_UploType uplo, Integer n,
                const Complex a[], Integer pda, double anorm, double *rcond, NagError *fail)
```

3 Description

nag_zpocon (f07fuc) estimates the condition number (in the 1-norm) of a complex Hermitian positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a call to nag_zhe_norm (f16ucc) to compute $\|A\|_1$ and a call to nag_zpotrf (f07frc) to compute the Cholesky factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

- 1: **order** – Nag_OrderType *Input*
On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order = Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.
Constraint: **order = Nag_RowMajor** or **Nag_ColMajor**.
- 2: **uplo** – Nag_UploType *Input*
On entry: indicates whether A has been factorized as $U^H U$ or LL^H as follows:
 if **uplo = Nag_Upper**, $A = U^H U$, where U is upper triangular;
 if **uplo = Nag_Lower**, $A = LL^H$, where L is lower triangular.
Constraint: **uplo = Nag_Upper** or **Nag_Lower**.
- 3: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.

- 4: **a**[*dim*] – const Complex *Input*
Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.
On entry: the Cholesky factor of *A*, as returned by nag_zpotrf (f07frc).
- 5: **pda** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.
- 6: **anorm** – double *Input*
On entry: the 1-norm of the **original** matrix *A*, which may be computed by calling nag_zhe_norm (f16ucc). **anorm** must be computed either **before** calling nag_zpotrf (f07frc) or else from a copy of the original matrix *A*.
Constraint: **anorm** ≥ 0.0 .
- 7: **rcond** – double * *Output*
On exit: an estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.
- 8: **fail** – NagError * *Output*
The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = *<value>*.

Constraint: **n** ≥ 0 .

On entry, **pda** = *<value>*.

Constraint: **pda** > 0 .

NE_INT_2

On entry, **pda** = *<value>*, **n** = *<value>*.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

NE_REAL

On entry, **anorm** = *<value>*.

Constraint: **anorm** ≥ 0.0 .

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter *<value>* had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to `nag_zpocon` (f07fuc) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to `nag_zpotrs` (f07fsc) with 1 right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is `nag_dpocon` (f07fgc).

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive-definite and must first be factorized by `nag_zpotrf` (f07frc). The true condition number in the 1-norm is 201.92.

9.1 Program Text

```

/* nag_zpocon (f07fuc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <naga02.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double  anorm, rcond;
    Integer i, j, n, pda;
    Integer exit_status=0;
    Nag_UploType uplo_enum;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    char    uplo[2];
    Complex *a=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);
    Vprintf("f07fuc Example Program Results\n\n");

```

```

/* Skip heading in data file */
Vscanf("%*[\n] ");
Vscanf("%ld%*[\n] ", &n);
#ifdef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif

/* Allocate memory */
if ( !(a = NAG_ALLOC(n * n, Complex)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A from data file */
Vscanf(" ' %ls '%*[\n] ", uplo);
if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
else
{
    Vprintf("Unrecognised character for Nag_UploType type\n");
    exit_status = -1;
    goto END;
}

if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        Vscanf("%*[\n] ");
    }
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        Vscanf("%*[\n] ");
    }
}

/* Compute norm of A */
f16ucc(order, Nag_OneNorm, uplo_enum, n, a, pda, &anorm, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f16ucc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Factorize A */
f07frc(order, uplo_enum, n, a, pda, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07frc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Estimate condition number */
f07fuc(order, uplo_enum, n, a, pda, anorm, &rcond, &fail);
if (fail.code != NE_NOERROR)

```

```

    {
      Vprintf("Error from f07fuc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
    if (rcond >= X02AJC)
      Vprintf("Estimate of condition number =%10.2e\n", 1.0/rcond);
    else
      Vprintf("A is singular to working precision\n");
END:
    if (a) NAG_FREE(a);
    return exit_status;
}

```

9.2 Program Data

f07fuc Example Program Data

```

4                                     :Value of N
'L'                                   :Value of UPLO
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A

```

9.3 Program Results

f07fuc Example Program Results

Estimate of condition number = 1.51e+02
